# **APPLIED MATHEMATICS, PHD**

The Applied Mathematics PhD program is a joint program with The University of Alabama in Birmingham and The University of Alabama in Huntsville.

### **Admission Requirements**

To be admitted for a graduate degree, students are expected to satisfy the general requirements of the Graduate School, as stated in the Admission Criteria section of this catalog. In support of the application, each applicant must submit scores on the general test of the Graduate Record Examination; the advanced portion is desirable but not required.

See the Admission Criteria section of this catalog for more information.

### **Doctor of Philosophy Degree in Applied Mathematics**

The Doctor of Philosophy degree in Applied Mathematics is a joint program with The University of Alabama in Birmingham and The University of Alabama in Huntsville. The program is intended as a research degree and is awarded based on scholarly proficiency (as demonstrated by course work, passing the Joint Program Qualifying Exam and the Comprehensive Exam) and the ability to conduct independent, original research (demonstrated by the PhD dissertation). A successful student must:

- Pass the Joint Program Exam (JPE), also called the Qualifying Exam. The Joint Program Examinations in Real Analysis and Linear Algebra are given during two periods each year (one in May and one in September). During each period a student may take one or both of the exams but subject to the following restrictions: (1) either exam may be attempted at most twice and (2) a student may participate in exams during no more than three periods. Core courses that will help students prepare for these exams are: MATH 580 Real Analysis I, MATH 681 Real Analysis II, MATH 572 Linear Algebra andMATH 510 Numerical Linear Algebra.
- Complete 54 semester hours of graduate courses. The grade of each course has to be at least a B. The student's supervisory committee and the Joint Program Committee must approve the selection of all these courses. At least 18 hours must be in a major area of concentration, selected so that the student will be prepared to conduct research in an area of applied mathematics, while at least 12 hours have to be in a minor area of study, which is a subject outside mathematics. (No courses counted towards an MA degree can be used. Also, the following courses do not count toward this degree: MATH 504 Topics Mod Math Teachers, MATH 505 Geometry: Secondary Teachers, MATH 508 Topics In Algebra, MATH 551 Math Stats W/Applictn I, MATH 552 Math Stats W/Applictn II, MATH 570 Prin Modern Algebra I,MATH 586 Intro Real Analysis I, MATH 587 Intro to Real Analysis II, and MATH 591 Teaching College Math.)
- · Pass a foreign language or tool of research exam.
- Pass the Comprehensive Exam, which consists of a written part and an oral part.
- Prepare a dissertation, which must be a genuine contribution to mathematics.
- · Pass the Final Examination (thesis defense).

For university rules regarding transfer credit, residency requirements, and other policies and deadlines, refer to the Academic Policies section of the Graduate Catalog, or see the Graduate Program Director.

#### **Course Work Requirement**

Students must complete 54 credit hours in order to qualify for the PhD. Study plans for students wishing to focus in Scientific Computing/PDE, Optimization, Statistics can be found in the student handbook.

The grade of each course has to be at least a B. The student's supervisory committee and the Joint Program Committee must approve the selection of all these courses. At least 18 hours must be in a major area of concentration, selected so that the student will be prepared to conduct research in an area of applied mathematics, while at least 12 hours have to be in a minor area of study, which is a subject outside mathematics.

#### Code and Title

Hours

## The 18 hours in a Major area of applied mathematics can come from any of the following courses:

	MATH 510	Numerical Linear Algebra	
	MATH 511	Numerical Analysis I	
	MATH 512	Numerical Analysis II	
	MATH 520	Linear Optimization Theory	
	MATH 521	Non-Linear Optimization Theory	
	MATH 541	Boundary Value Problems	
	MATH 554	Math Statistics I	
	MATH 555	Math Statistics II	
	MATH 557	Stochastic Processes I	
	MATH 559	Stochastic Processes II	
	MATH 610	Iteratve Meth Linear Sys	
	MATH 611	Numerical PDEs	
	MATH 642	Partial Differential Equations	
	MATH 644	Singular Perturbations	
	This is not an exhaustive list courses options. Other options may include from The University of Alabama in Huntsville and The University of Alabama in Birmingham may be selected with advisor approval.		
h	he 12 credit hours in a minor area of study can come from		

#### The 12 credit hours in a minor area of study can come from graduate level courses offered by the Departments of Physics, Computer Science, Aerospace Engineering and Mechanics, Chemical and Biological Engineering Economics or Applied Statistics.

Additional courses are available to students that provide the foundation to do research at the PhD level. Students with an uneven preparation at the undergraduate level may be advised to take foundation courses before proceeding with the program above. For example, students in the Ph.D. program may be initially advised to take the Master's level analysis course, MATH 587 Intro to Real Analysis II, before taking MATH 580 Real Analysis I.

Because a doctoral degree usually requires five years of full-time study, financial support is provided for five years, with the possibility of a sixth year of support. A typical course load is three courses per semester. If a student is employed as a Graduate Teaching Assistant (equivalent to a 6-hour teaching load), the minimum course load is 6 hours. However, the total course load plus teaching must be between 12 to 18 hours inclusive.

#### **Time to Complete**

Coursework may be finished within two years after the Qualifying Exam. Research should be started while coursework is still underway. Typically, work on the thesis itself takes 12-18 months. Therefore, depending on your background, it can take four to six years to obtain both the M.S. and the Ph.D. degree.

#### Joint Program Examinations

Every student planning to earn the PhD in Applied Mathematics must pass the two Joint Program Examinations. One exam covers real analysis. The other exam covers linear algebra and numerical linear algebra. Each exam is three and one half hours long.

The exams are administered twice a year. During each administration, a student may take one or both of the exams. A single exam may be attempted at most twice, with a maximum of three attempts allowed for passing both exams.

Any student considering taking this examination should meet as soon as possible with the Department Chair and Graduate Program Director.

#### **Topics in Real Analysis**

- Lebesgue measure on R<sup>1</sup>: outer measure, measurable sets and Lebesgue measure, non-measurable sets, measurable functions.
- The Lebesgue integral in R<sup>1</sup>: positive functions and general functions, comparison with the proper and improper Riemann integral.
- Differentiation and integration: monotone functions, functions of bounded variation, absolute continuity, the fundamental theorem of calculus.
- Definition of a positive measure, measure spaces, measurable functions, the integral with respect to a positive measure.
- Convergence theorems for positive measures: monotone and dominated convergence.
- $L^p$  spaces for positive measures with  $p=1,2,...,\infty$ , definition, completeness.
- Product measure, Lebesgue measure on R<sup>k</sup>, Fubini's theorem.

#### **Topics in Linear Algebra**

- Vector spaces over a field: subspaces
- · quotient spaces
- complementary subspaces
- · bases as maximal linearly independent subsets
- finite dimensional vector spaces
- linear transformations
- null spaces
- ranges
- invariant subspaces
- vector space isomorphisms
- matrix of a linear transformations
- · rank and nullity of linear transformations and matrices
- change of basis
- · equivalence and similarity of matrices
- dual spaces and bases
- diagonalization of linear operators and matrices
- Cayley-Hamilton theorem and minimal polynomials
- Jordan canonical forms
- · real and complex normed and inner product spaces
- · Cauchy-Schwarz and triangle inequalities
- · orthogonal complements, orthonormal sets

- · Fourier coefficients and the Bessel inequality
- adjoint of a linear operator
- positive definite operators and matrices
- · unitary diagonalization of normal operators and matrices
- · orthogonal diagonalization of real, symmetric matrices